FRICTION FACTOR FOR FLOW OF A FLUIDIZED BED IN AN AIRSLIDE

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Received April 22nd, 1975

A relation for calculation of friction factor of fluidized beds was derived at the assumption of an existing analogy between the liquid flow in open channels and flow of a fluidized bed in airslides. The dependence of the friction factor on flow conditions was generalized in a similar way as with liquids. This was possible due to generalization of the relative viscosity of fluidized beds. Original experimental data together with those published in literature were evaluated by use of relative viscosity of the bed calculated from the dependence on relative gas velocity (ratio of gas velocity to the minimum fluidization velocity) as well as on the dimensionless velocity (ratio of the difference of fluid velocity and minimum fluidization velocity to the difference between the minimum elutriation velocity and the minimum fluidization velocity). Better results were obtained by use of the viscosity of the fluidized bed characterized by the dimensionless velocity.

This study is a part of a wider research on flow of a fluidized bed through airslides. In previous studies of this series the apparatus was described and the literature survey was published together with the experimental results[1,2] used for calculation of the relative viscosity of the fluidized bed[3]. Here an attempt has been made to obtain a relation for calculation of the friction factor of the fluidized bed on a wide range of conditions. Our contribution is based on an analogy between the flow of Newtonian fluids in open channels and flow of the fluidized bed in airslides which has been proved as satisfactory[3]. Equations based on this assumption should have a wider range of validity than various empirical relations[4–7] or other relations[8–13] which are based on assumptions that cannot be verified.

It can be expected that especially empirical relations[4–7] will have a validity limited to experimental conditions under which the experiments were performed. Thus we conclude that validity of equations based on the dimensionless analysis[4,7] is also limited. In this case the results were obtained by measurements with a very fine-grained material in an apparatus with a distributor formed by porous ceramic plates. Validity of these relations for different conditions is disputable (*e.g.* for coarse particles and multiorifice grids).

Other authors[8] have made an attempt to apply the hypothesis on existing analogy between the flow of liquids and the flow of the fluidized bed. The relation describing

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the flow of the fluidized bed is derived on basis of the assumption that the resistance to its flow in the airslide is the same as to its flow through the open channel. But an error has been made which is discussed in one of our recent studies[2]. Only a part of the results can be taken as the verification of their assumptions.

In other publications[9-13], in which the hypothesis is also based on the existing analogy between the flow of the fluidized bed and flow of liquids, are for description of the fluidized bed flow directly used relations which are characterizing the liquid flow. Other authors[11,12] with the reference to literature[13] are determining the mass flow rate through an airslide by the use of relation for turbulent flow of liquids in open channels. On the contrary Siemes and Hellmer[9,10] are applying to the fluidized bed the relation suitable for laminar flow of liquids through open channels. They consider the flow of the fluidized bed as laminar on basis of visual observations of the velocity profile of particles on the surface of the fluidized bed and according to the value of the Reynolds number.

THEORETICAL

Our solution of the problem of the fluidized bed flow in an airslide is based on the assumption of the existing analogy between the flow of the fluidized bed in an airslide and flow of liquids in open channels[3,14]. Friction factor of the fluidized bed is calculated by use of the relation analogical to the relation for expressing the mass flow rate of liquid in open channels in steady uniform flow:

$$\dot{m}_s = bh\varrho_B(8\,g/f_B)^{1/2}\,(r_h \sin\beta)^{1/2}\,, \quad r_h = (bh)/(b + 2\,h)\,. \qquad (1), (2)$$

Dependence of the friction factor f_B on flow conditions is expressed in the form[3,14]

$$f_B = BR_B^a\,, \quad R_B = \mathrm{Re}_B\eta_B\,, \quad B = A/\eta_B^a\,. \qquad (3)-(5)$$

As viscosity of the fluidized bed is changing with the velocity of the fluidizing fluid[9,10,15-21] the quantity B is – with regard to Eq. (5) – also a function of velocity. Thus for various experimental conditions the set of Eqs (3) is obtained.

The set of Eqs (3) should be expressed by a single relation through introduction of quantities B_O and R_O which are determined by relations

$$B_O = A/\eta_{BO}^a\,, \quad R_O = \mathrm{Re}_B\eta_{BO} = R_BE_B\,. \qquad (6), (7)$$

The relative velocity W and the dimensionless velocity U of the fluidizing fluid are defined by relations $W = w/w_p$ and $U = (w - w_p)/(w_e - w_p)$.

The relation (3) can be then rearranged in the form

$$f_B = B_O R_O^a \,, \tag{8}$$

where $B_O \neq B_O(W)$ or $B_O \neq B_O(U)$.

Eq. (*8*) can be used for calculation of the friction factor f_B if the quantities a, B_O and the concrete form of the relation $E_B = E_B(W)$ or $E_B = E_B(U)$ are known.

Relations for expression of relative viscosity are given by Eqs[3,14]

$$1/E_B = 0.403\{(W - 1.21) + [5.049 + (W - 1.21)^2]^{1/2}\} - 0.563 \,, \quad [W_O = 2.5] \,, \tag{9}$$

$$1/E_B = 6.042\{(U - 0.014) + [0.022 + (U - 0.014)^2]^{1/2}\} - 0.563 \,, \quad [U_O = 0.1] \,. \tag{10}$$

Values of constants a and B_O can be determined by the regression analysis of experimental data.

We have paid in our study attention to lower fluidized beds which are from above limited by the so-called critical height h_{cr}. According to the author[5], in the neighbourhood of this critical height the transition of the flow regime takes place. The critical height is a function of several variables.

With the quantity R_O introduced, we consider that it is more suitable to express the given limiting condition by use of the critical quantity $R_{O,cr}$ which is assumed to be a simpler function than the value h_{cr}. A simple relation is obtained under the assumption that at flow of the fluidized bed a change in the flow character takes place, as with liquids, at a constant value of the Reynolds number $Re_{B,cr}$

$$Re_{B,cr} = 4r_{h,cr}\bar{u}_s\varrho_B/\eta_B \,, \tag{11}$$

where $r_{h,cr}$ is the critical value of the hydraulic radius which corresponds to the critical height h_{cr}. The critical value $R_{O,cr}$ is then introduced by the relation

$$R_{O,cr} = Re_{B,cr}\eta_{BO} \,. \tag{12}$$

As we have considered $Re_{B,cr}$ = const and $\eta_{BO} \neq \eta_{BO}(W)$, or

$$\eta_{BO} \neq \eta_{BO}(U) \quad \text{it holds} \quad R_{O,cr} \neq R_{O,cr}(W) \quad \text{or} \quad R_{O,cr} \neq R_{O,cr}(U) \,.$$

But as the value η_{BO} depends on the type of granular material the value $R_{O,cr}$ also differs for various materials.

DISCUSSION

Our conclusions were verified by our own experimental data which have been published in one of our recent publications[2]. There were also given basical properties of the materials used *i.e.* of corundum, glass balotine and sand together with the description of the experimental apparatus and of the experimental methods applied.

For determination of constants a and B_O in Eq. (*8*) the dependence of viscosity E_B on velocity W – Eq. (*9*) or on velocity U – Eq. (*10*) were used. In the case of corundum the available experimental values of E_B were used. Quantity R_O was then calculated from values E_B and R_B, according to the relation (*7*). Additional data on flow of a bed of crystal sugar in the airslide[5] were used together with the data mentioned above. Also data from studies[9,10] on flow of the bed of fine sand were used for comparison (the data which were not used in the calculation of constants a and B_O).

The values f_B and R_O which were summarized in studies[22,23] were obtained from experimental data by use of the relative velocity W for calculation of the relative viscosity E_B'. The regression relation was obtained from these data by the least square method[24]

$$f_B = 24{\cdot}74/R_O^{1.35}\,, \qquad (13)$$

where R_O has the unit [kg m^{-1} s^{-1}]. This equation is graphically plotted in Fig. 1 together with the experimental data and the limits of confidence. There are also plotted data recalculated from literature[9,10]. The calculated values are summarized in our recent studies[22,23]. The total number of 800 experiments included into this

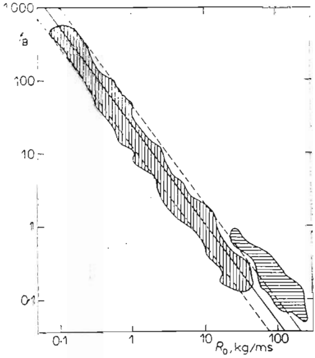

FIG. 1

Friction Factor f_B in Dependence on Quantity R_0 Determined by use of the dependence $E_B(W)$

——— According to Eq. (*13*), - - - - confidence limits. Vertically cross-hatched our data (corundum, coarse sand, balotine) and sugar[5], horizontally cross-hatched fine sand[9,10].

graph is considerable. Thus for illustration are, instead of individual experimental points, plotted cross-hatched areas together with regions of our own experiments with corundum (286 experiments) with coarse sand (52 experiments) and with balotine (176 experiments) and of results of other authors[5] with crystal sugar (24 experiments) and the region of results[9,10] obtained with fine sand (162 experiments).

As it is obvious, the points from literature[9,10] differ considerably from the region for other materials. This could be first of all the result of small particles of the used sand (equivalent diameter 0·2 mm) in comparison with greater sizes of other materials. Then the difference in ratios of incipient elutriation and of incipient fluidization velocities is considerable and the use of relative velocity W as independent variable is already problematic[3]. According to our assumptions the velocity U should be more suitable in such case as the independent variable.

From the values f_B and R_O obtained by use of the quantity U (it's values are given in our recent publications[22,23]) we have obtained by the least square method[24] the regression equation

$$f_B = 23{\cdot}12/R_O^{1.36}\,, \tag{14}$$

which deviates only slightly from Eq. (*13*). The plot of Eq. (*14*) together with the confidence limits and with the evaluated data is given in Fig. 2. Also there are plotted for comparison the data by Siemes and Hellmer[9,10] which were recalculated by use of the quantity U. It is obvious that these data unlike the already mentioned disadvantage at the use of quantity W, fit very well with the other data included into the regression analysis. This proves a greater universality of the dependence $E_B(U)$ that we have expected.

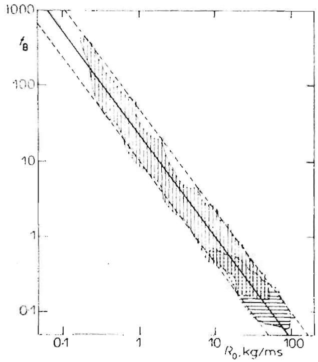

FIG. 2

Friction Factor f_B in Dependence on Quantity R_0 Determined from the Dependence $E_B(U)$

——— According to Eq. (*14*), ----- confidence limits. Cross-hatched regions are denoted in the same manner as those in Fig. 1.

But application of the dimensionless velocity U is causing a slightly greater spread of data around the regression straightline (see Fig. 2). This can be explained by the already mentioned[3] greater error in determination of the velocity U in comparison with the error in determination of W.

In both equations (*13*) and (*14*) we have obtained somewhat greater value of exponent a than the boundary value for liquid flow (the value one is reached at laminar flow). In our opinion this is due to the existence of an immobile layer which obviously appears between the holes of the grid[1]. This assumption could be supported by the values of the exponent equal to one which was obtained by measurements with balotine[2]. With respect to the spherical shape and smooth surface of particles of balotine, a small height can be expected of this immobile bed. Beside this, in our recent study[2] the actucal bed height was corrected by use of the so-called static holdup. Such corection is obviously justified. But there is a problem of a suitable method for determination of static holdup.

As we have already mentioned earlier the experimental data were used for evaluation of Eqs (*13*) and (*14*) for which the height of the fluidized bed h was lower or at the maximum equal to the critical height h_{cr} *i.e.* for which held $h \leqq h_{cr}$. We have made an attempt to express this condition by use of the quantity $R_{O,cr}$ defined by Eq. (*12*). The limiting values $R_{O,cr}$ are given in Table I. As with corundum the critical height has not been found in the range of measured data, the value given for $R_{O,cr}$ in Table I is the largest calculated value. This value is thus smaller or at the maximum equal to the actual value $R_{O,cr}$. The data given in Table I must be considered orientation values. The more reliable information for higher fluidized beds could not have been obtained due to small number of available data.

TABLE I

Values of $R_{0,cr}$ for Corundum, Sand, Balotine and Sugar[5]

Material	β degree	W	h_{cr} mm	$R_{0,cr}$ kg/ms
Corundum	3	1·7	42	42·6
Coarse sand	3	1·7	34	4·4
Balotine	3	2·0	25	12·2
	5	2·0	24	13·2
	6	2·0	26	18·5
Sugar[5]	0·5	1·8	28	1·6
	1	1·8	28	2·4
	2	1·8	25	2·6
	3	1·8	29	5·1

Application of the given relations is limited to low fluidized beds. On basis of the value f_B the mass flow rate can be thus calculated for various materials and industrial airslides can be designed especially for cooling or heating and classification *i.e.* for cases when the use of low beds is advantageous.

LIST OF SYMBOLS

a	exponent in Eq. (*3*)
A	constant in Eq. (*5*)
b	width of grid of airslide
B	quantity defined by Eq. (*5*)
B_O	value of quantity B at reference velocity of the fluidizing fluid $W = W_O$ or $U = U_O$
E_B	relative viscosity of the fluidized bed
f	friction factor
g	gravitational acceleration
h	height of fluidized bed at uniform flow
$\dot{m}_s$	mass flow rate of solid particles
r_h	hydraulic radius
R_B	quantity defined by Eq. (*4*)
R_O	quantity defined by Eq. (*7*)
Re	Reynolds number
$\bar{u}_s$	mean velocity of granular material
U	dimensionless velocity of fluidizing fluid
w	superficial velocity of fluidizing fluid
w_e	superficial velocity of fluidizing fluid at the minimum elutriation velocity of solid particles
w_p	superficial velocity of fluidizing fluid at the minimum velocity of fluidization
W	relative velocity of fluidizing fluid
β	angle of inclination of the airslide grid
η	dynamic viscosity
ϱ	density

Subscripts

B	related to the fluidized bed
cr	critical
0	reference

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Translated by M. Rylek.